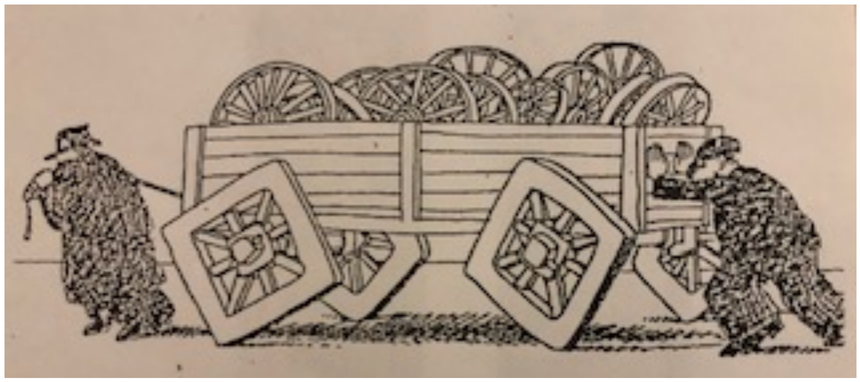
Chapter 1
"Paradigms"



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How many queens can you have on a chessboard?

It's a good question for parties, but even more illuminating at large family gatherings, for the proffered answers eventually range somewhat farther than you might think.

A typical early answer is 8. Sometimes this is because the proponent is thinking of the well-known puzzle of placing 8 queens on a chessboard such that none attacks any other, and sometimes just because there are 8 pawns to be promoted. At this point numbers like 16 (both sides), then 18 (each side started with one queen) are offered. Then comes 17, "wait, somehow you have to get the opposing pawns out of the way." If the discussion gets this far, the host may very well pull out a chessboard (or two) and let people start testing their ideas. This is the typical flow at a party.

For family gatherings, however, you might very well hear "64" shouted out with confidence by one of the youngsters. Why? *Because there are 64 squares on the chessboard.* Oh, the adults were in a box, making the hidden assumption that the challenge required the rules of play to be enforced. Out of the mouth of babes…

But my favorite answer is 128.

Are we really restricted to just one piece per square? Another box. Just how many pieces could one cram onto a chessboard? Out come the rest of the family chess sets, the large ornate boards, the conventional competition boards, the small portable boards, even the tiny, magnetized travel boards. (Ok, perhaps my family was unusual.)

For most puzzles, one expects there to be one right answer – and only one. The challenge, the fun, and even the pedagogical significance of the puzzle lies in the obstacles that must be surmounted in order to find this one right answer. This confining view is strengthened by the modern education culture, problems with only one right answer are much easier to grade.

Paradigm puzzles are different. The objective is to uncover hidden assumptions behind each theory and generate as many valid answers as you can justify using the tools of logic, proof, reason, and mathematics. It requires a deep well of curiosity, the persistence to experiment, and the courage to let paradigm and dogma duke it out. The goal is not to solve the puzzle per se, but to gain *understanding*, to more completely grok the phase space of puzzles.

Paradigm comes from the Greek word for pattern. It was given its modern meaning by Thomas Kuhn in his 1972 seminal work, "The Structure of Scientific Revolutions." For the Intrepid Reader, this book is your first homework assignment.

A paradigm is the set of assumptions behind a particular worldview. We are hardwired by evolution to deal with complexity via paradigms. A relevant paradigm allows us to quickly assess what is important or threatening, and therefore what can safely be ignored. In a sensory overloaded environment, the need to reduce the cognitive load is paramount. Computation is expensive; our minds are finite, so paradigms offer optimized heuristics that allow us to navigate life efficiently, successfully, and safely. A well-formed paradigm allows us to see the next interesting problem, provides well validated guidelines on methods of attack with high probabilities of success, metrics to determining when the problem has been successfully solved, and confidence that the problem is finally, definitively, and irrefutably resolved.

But.

They also cage us. Strong paradigms can distort our perceptions to the point that we fail to perceive non-conforming data (Kuhn provides intriguing, occasionally disturbing, examples). Even scientists. Particularly scientists since their profession requires mastering *very* strong paradigms. In Kuhn's model any scientific discipline spends most of its history in a normal mode where the paradigm is stable, advances happen rapidly, and solutions are checked off with a predictable expenditure of resources. However, hiding among the long list of successfully solved problems there begins to accumulate a small set of problems that resist solution. No one expected them to be intractable, yet as the decades go by even the combined assault of the field's luminaries fail to resolve them.

These intractable problems are the clues to the next paradigm, a larger paradigm that subsumes the current one – nested boxes. In common parlance, the act of breaking out of the current paradigm is called, "thinking out of the box." It is a good metaphor. A common mischaracterization of Kuhn's work is that paradigms are a shackle, an obstacle to sound thinking – not so, at least not most of the time. A mature paradigm is the most effective tool we have for uncovering the clues that point to the enclosing paradigm, the next larger box. They become an obstacle to understanding only when held onto for too long, and it is not a trivial matter to know when that point occurs. Historically, the best clue seems to be the emergence of a small set of intractable problems that everyone expected to be solved in due time without much fuss.

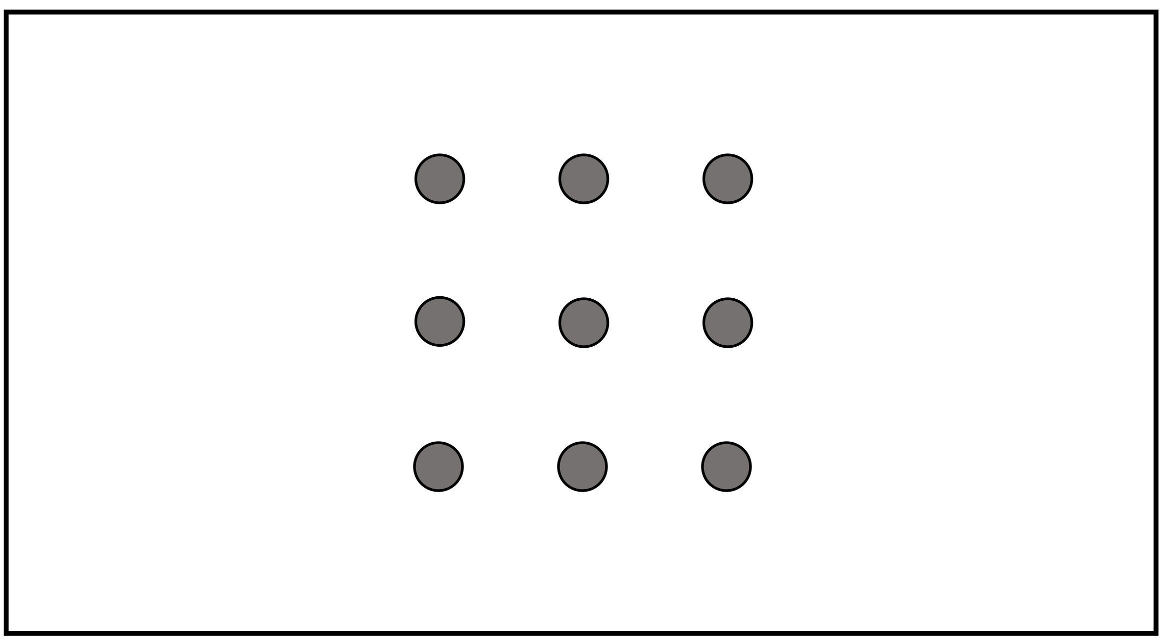
There are several such clues in quantum physics, and in due course they will be considered. However, one is of such fundamental importance, and has gone unresolved for so long, that it will provide the pinion of this book, the measurement problem.

Surprisingly, physicists don't know what causes a quantum system to collapse from a superposition of states to a single classical state. It's a case of, "We know it when we see it." The mathematics permits us to claim it occurred just about anywhere along the observation chain and the different computations all yield the same answer. It is a conundrum.

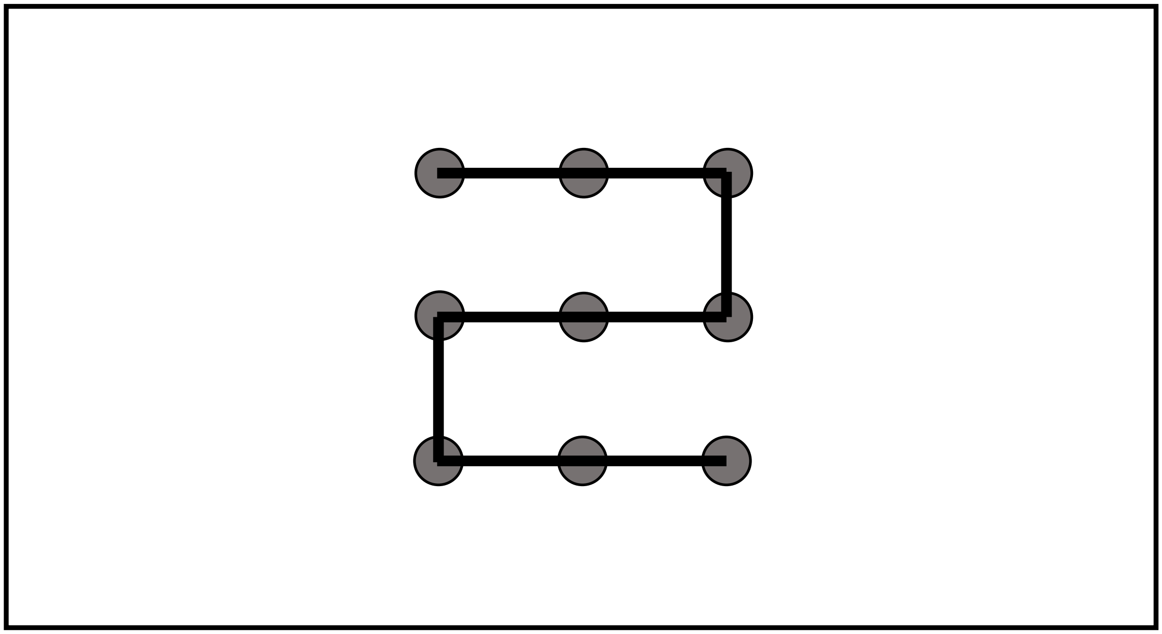
It's tempting to suggest that all we need is to think out of the box, but these conceptual boxes, these paradigms, are very resistant to questioning, and if professional scientists have their heroes, most of them are those historical figures who did indeed make contributions that broke the current paradigm. We'd all like to be paradigm pioneers.

So, thinking out of the box is more simply said than done.

To see a little more clearly how challenging it is to break an existing paradigm, and to gain a deeper understanding of the full nature of paradigms, here is a well-known puzzle designed to help us think out of the box. It's called the 9 Dot Puzzle, and it consist of 9 dots arranged in a regular 3x3 grid, a square of dots. The challenge is to connect all the dots but under constraints typical of puzzles, games & sports. In this case, connect all 9 dots with just 4 straight lines, never allowing the writing instrument to lose contact with the surface.



It's easy to do with 5 lines, here's one solution.



Can you think of others?

But now try it with just 4 lines. If you've seen this puzzle before, and know what's coming, be patient; we'll get to you. If this is your first exposure to this puzzle, take a moment to try and solve it (don't turn the page). It's good training in paradigms, how to recognize when the one you are in is hampering your thinking and blocking the path to a solution. Other terms for escaping the existing paradigm which you may have heard are:

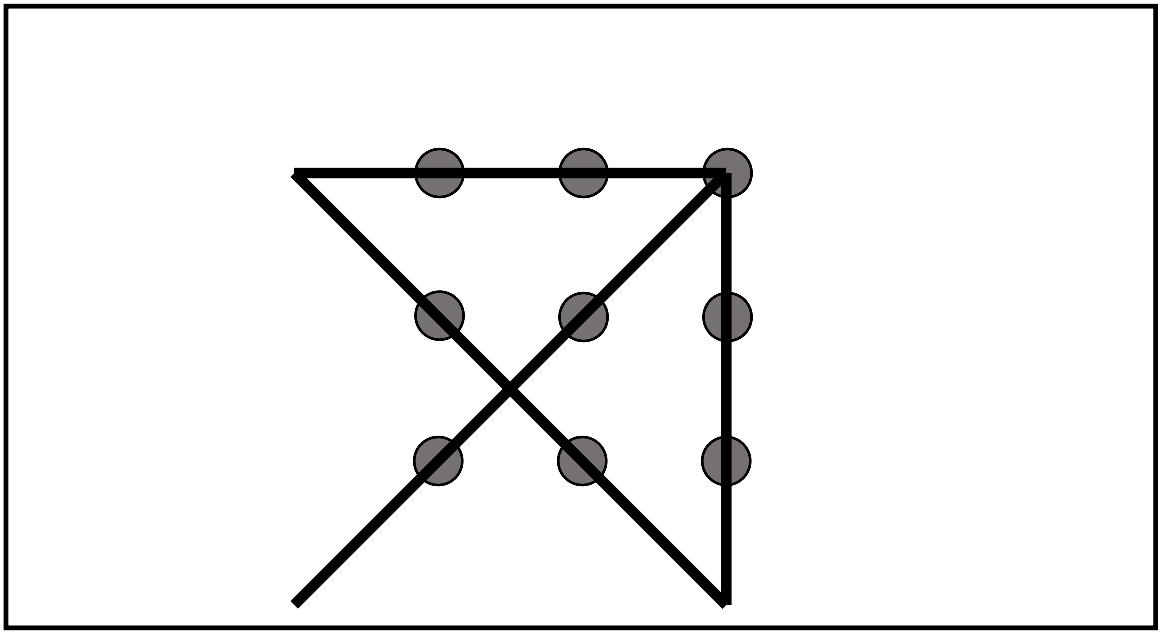
"Thinking out of the box" – current idiom phrase in Western culture.

"Lateral thinking" – great book with the same title by the way.

"Unconventional thinking" – used a lot in the Start Trek franchise.

How'd you do?

So here is the solution.



Ahhh! Truly out of the box. That 3x3 grid of dots, that perfect square, does in fact form a box, a perceptual box. The human visual system automatically divides the space into the square as foreground and the rest as background. To divine the solution requires letting go of that perceptual bias.

However, you are always in a box. There is always a paradigm that is guiding (and unavoidably limiting) our thinking. To see that we are going to repeat the 9 Dot Puzzle, but with one change – now do it with just 3 straight lines.

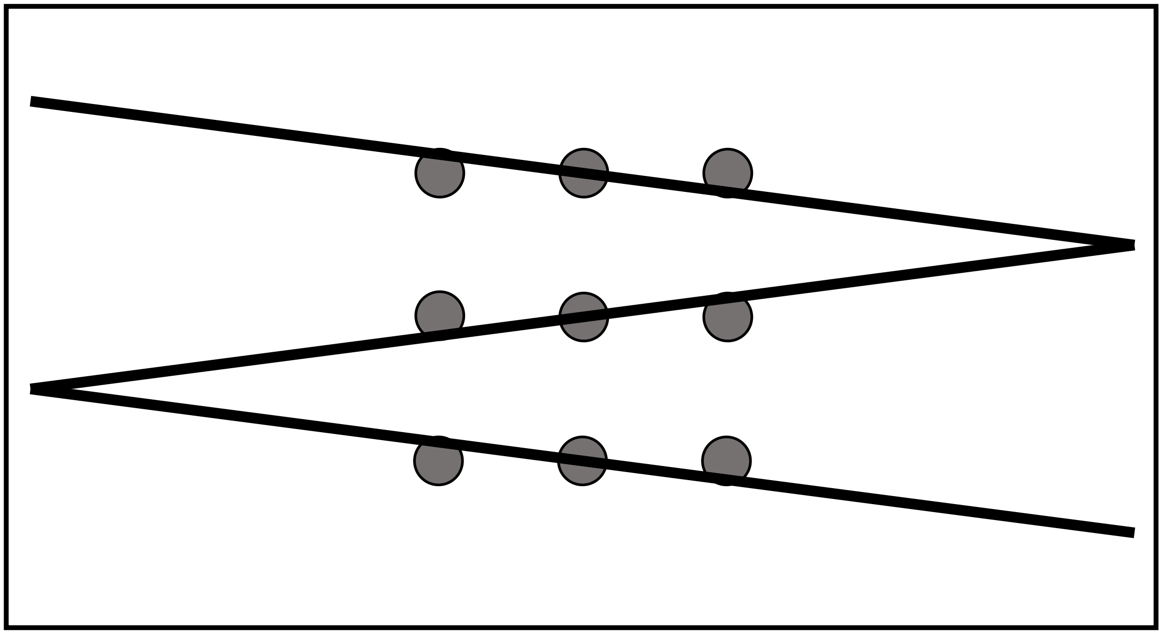
Gotcha. I promised we'd get to those of you who have seen this puzzle before.

But surely this is an impossible constraint. Parallel lines never meet. So seriously, grab a piece of paper. Draw your own set of 9 dots and see if you can connect them with just 3 straight lines. Grab a pen or pencil and spend a minute or two trying things. Be creative. And no, you are not allowed to lift your writing instrument off the surface; just a reminder of one of the constraints of the puzzle. Once you start drawing a straight line complete it and continue with the next straight line.

The point of this variation is to drive home the idea that you are always in a box. Even after successfully thinking out of the box, you are still in a box, but the new box will be different. It's not sufficient to try to draw your lines even farther from that square box of 9 dots, parallel lines never meet no matter how long you make them. The paradigm you are in now is different, it's not the paradigm forced upon you by the human perceptual system. It is a different paradigm, one you learned in school, and one that has been hinted at, alluded to twice in fact in the text which introduced this variation.

How'd you do?

Here's the solution.



Start the first line far, far away, drawing it through the 1st dot near its top edge, through the center of the 2nd dot, and through the 3rd dot near its bottom edge; then extend it out far enough that when the second line is started it can do the same for the 6th, 5th & 4th dots, and finally repeat for the third line through the 7th, 8th & 9th dots.

Now "wait a minute" you say, "that's cheating." Not actually, just devious. It wasn't the 9 *Point* Puzzle; points have no extent. But if you have a good education that included geometry, then the box that makes this solution hard to see is the *geometry* box – "parallel lines never meet."

That's the point of the 9 Dot Puzzle – you're always in a box. Getting out of the first one is usually of little help in getting out of the second one. That was why the character, Paradigm, in the Paradigm Discourse "Boxes" so appreciated Understanding's gift of nested dolls – you're always in a box.

But I'm not convinced you believe me. Since one of the fundamentals of teaching is repetition, we're going to introduce another variation on the 9 Dot Puzzle. So, grab another piece of paper, draw a fresh 3x3 grid of 9 dots, and connect them again, but this time with just 1 straight line.

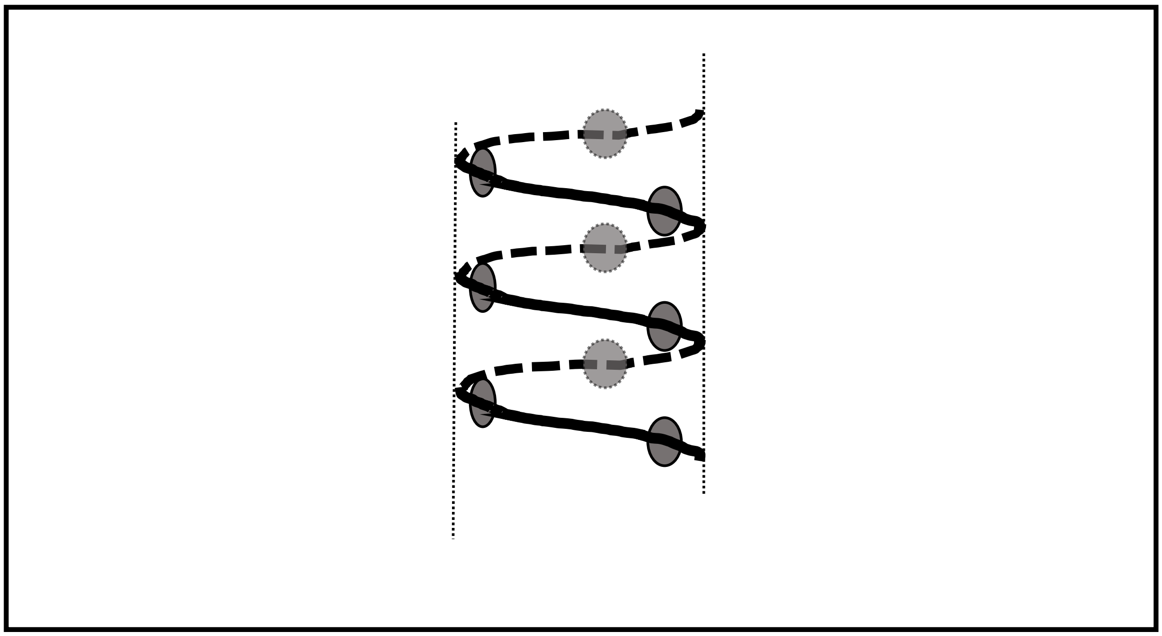
That's right, just *one* straight line, all 9 dots.

I can hear you complain from here, several years in the past, "That's impossible." Well maybe, but you have two examples now how hidden assumptions block our thinking. Perhaps a review is in order. The first box was one bequeathed us by evolution; the human perceptual system that divides our field of view into foreground and background. The second one was bequeathed to us by 8th grade geometry teachers everywhere; Proof and Logic would be just as upset by the above solution as you are.

So, what hidden assumptions are thwarting your path to a solution? Can you divine what they are? Be introspective, why do you believe a solution with 1 straight line is impossible? Oh, you may also want to consider the possibility that that Pesky Author you are reading right now gets a cheap thrill out of being devious, has he left clues (like he did before) that suggest what those hidden assumptions are?

How'd you do?

Here's one solution.



Take your piece of paper and roll it into a cylinder, tight enough that the distance around the cylinder between the left dots (1, 4, 7) and the right dots (3, 6, 9) is the same distance as between them and the center dots (2, 5, 8). Now slide the edges of the cylinder in opposite directions so the 9 dots all align on a spiral. Now draw the spiral.

"Hey," you complain, "that's not a straight line. You're cheating, again." No doubt the character Dogma would agree with you. But, no, it's not cheating, and it isn't even being devious. Actually, it is a straight line, but *straight* is now constrained by the manifold of a cylinder. The surface of a cylinder is curved in a higher dimension. It is the straightest possible line in the direction the spiral specifies. Any other line that connected the dots would be *longer*. That's the definition of a straight line, the shortest distance between two points.

This paradigm shift took the human species a long time to achieve. This particular paradigm barrier is called Euclidian Geometry, and it reigned supreme for 2,000 years before finally being broken. The non-Euclidian geometries (there are two) are applicable to curved surfaces, such as a cylinder in the solution above. The surface of a sphere is a good visual analog for one of the non-Euclidean geometries (positive), a saddle for the other (negative). Non-Euclidian geometries are also applicable to less regularly curved surfaces and serves as the mathematical foundation behind general relativity.

So, to reiterate again (sorry about the pun…, well, no not really), you're always in a box.

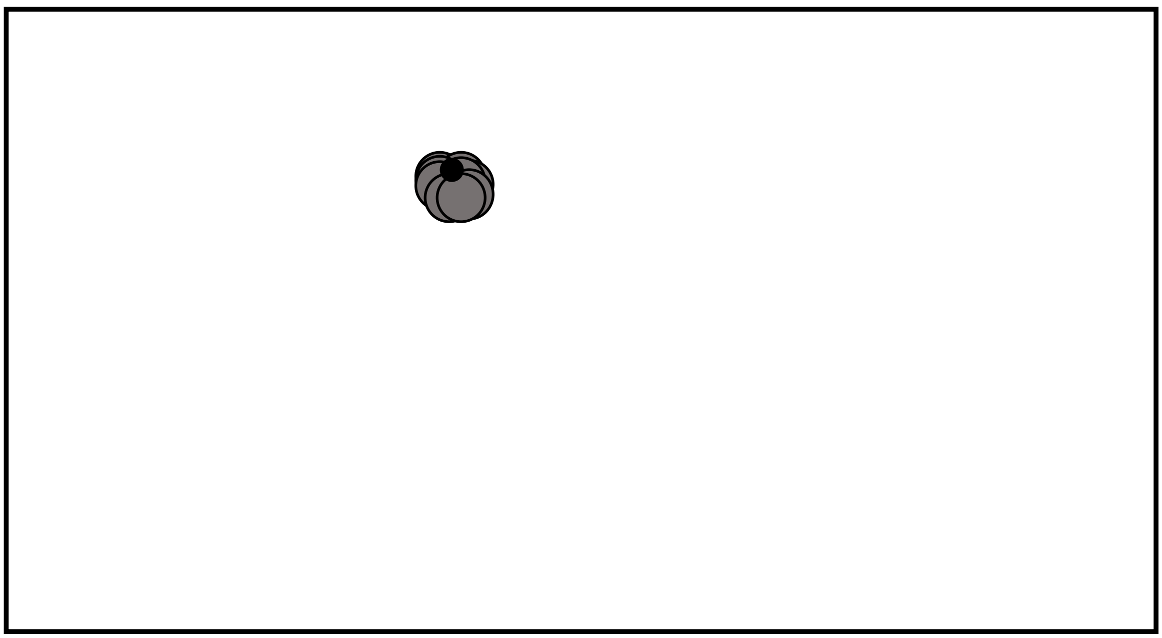
But I'm still not convinced you believe me. It is a bold claim to assert that you are always in a box. Therefore, get another piece of paper, draw the 9 dots and then solve the puzzle one more time. This time with no lines at all!

Admittedly this one is a bit harder; 0 lines, are you kidding me? Reason is shaking his head, but Mathematics, he's smiling. So perhaps a few clues are in order. What has been the pattern of the boxes so far?

5->4) Perceptual
4->3) Geometric
3->1) Non-Euclidean geometry
1->0) Hint (Topology)

How'd you do?

Here's one solution.



Fold the paper with a forward and a backward fold, so the lower row of dots overlaps the middle row and repeat so that the middle and third rows overlap the first row. If you stop here and use a writing instrument that draws a fat enough line, you have a second solution to the 1-line problem. Yes, a line is geometric idealization, but the drawing of a line always has width. For those of you for whom if 2 is good 3 is better, there is also the paint roller solution. Now you don't have to fold at all. To continue, repeat the horizontal folds but do them vertically. You now have all the dots overlapping each other. Simply stab the overlap with your writing instrument. No lines at all.

The fundamental point is to drive home the concept that we are always in a box, no matter how far we have come in understanding nature, that understanding is still framed by a paradigm. You might think this is stretching a point (pun alert), yet as our paradigms have become more encompassing, they have served to deepen our understanding of the universe and allowed the creation of marvelous technologies.

Overcoming this particular paradigm barrier led to the field of topology. Topology tries to answer the question, "what are the invariant properties of a space when extreme deformations are allowed?"

Alternatively, you could just crumple the sheet of paper into a ball and stab it – call this the black hole solution.

Looks like we are on a roll; generating not just one solution to the next problem, but multiple solutions. With a little bit of practice, it is possible to improve our ability to think out of the box. It is not just an innate skill that some have, and some don't. Whatever your current level of ability at recognizing the existing paradigm, and breaking free from it, it is possible to get better.

Therefore, take a second, a short pause from reading; and see if you can anticipate what the Pesky Author is going to ask next. You're always in a box. So, what is the next question?

Can you solve the puzzle with less than no lines at all?

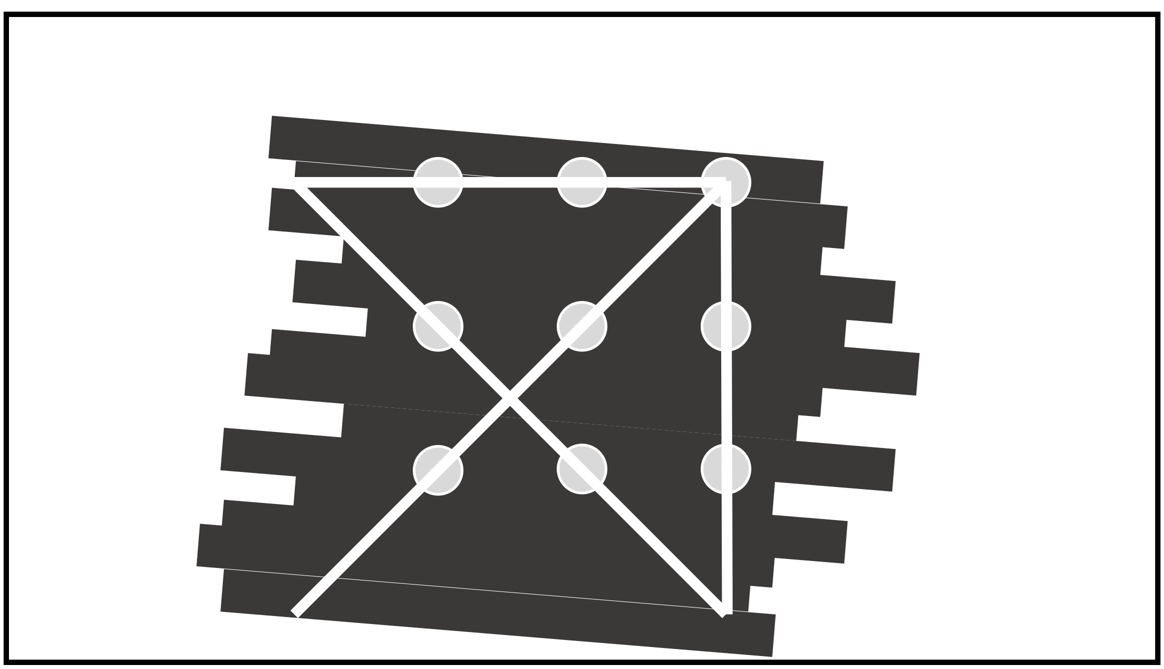
Did you see that coming? How can you have a solution that requires the existence of less than nothing? Perhaps you've had negative experience with a checking account, or at least have a passing familiarity with the number line.

Yes, now you understand the problem, and why it is actually a well-formed challenge, use a negative number of lines.

If you are stuck with paper, find a pencil. If you have access to a whiteboard, even better. But best is one of the old-fashioned chalk boards.

How'd you do?

Here is the solution.



Turn the piece of chalk on its side and rubbing it lengthwise against the board, create a solid region on the chalkboard big enough to hold the 9 Dot Puzzle. Now grab the eraser and mark out the 3x3 grid of 9 dots. Use the eraser to draw, say four lines in the arrow shape of the first out of the box solution. QED.

Call this the anti-matter solution.

You may be thinking this is beyond the pale, that we have left the spirit of the original puzzle far behind. However, this solution is a good analog to one that occurred in physics, and not that long ago. When a physical system is modeled with mathematics, it is often the case that the math admits more than one solution. For instance, the ballistic equation is quadratic, it admits two solutions; one which describes the trajectory of a cannon shell through the atmosphere, the other through the solid substance of the earth. Clearly, no soldier is going to use the second solution. The lesson we try to provide the student is that when one solution is 'non-physical' it should be ignored, and part of the training of a physicist is to develop in them an intuition for nonsensical solutions.

But this can lead to another paradigm barrier. When Dirac first solved the relativistic quantum equation for the electron, he found himself with two solutions. One described a particle with negative charge, and this he correctly connected with the electron, but the other mathematical solution described a particle with exactly the same mass and spin, but with a positive charge. Was the second solution non-physical? Dirac could find no good reason to disregard this solution, so he boldly predicted a new particle, which is now called the anti-electron (or positron as that is easier to pronounce) and generalized this single data point to a whole new class of particles, anti-matter. Experiments were promptly designed and run, and lo and behold, the new particle was observed in the tracks of cloud chambers. Even more significant is that now that we knew this particle was real, it was rediscovered in old cloud chamber photographs, ones taken years *before* Dirac's theory was published. The data had been there all along, but we were blind to it, because the paradigm of particle physics pre-Dirac did not include anti-matter. The positron tracks were ignored, regarded as noise in the experiment or bad experimental technique; a classic example of the reigning paradigm literally distorting the data to fit our expectations.

At this point your Pesky Author is gaining some confidence that you are buying the theme that you're always in a box. But I want to make sure, so challenge time; can you figure out what the next question is going to be?

Can you connect all the dots with half a line? Or three and a half lines; I'll take any fractional number of lines as a solution.

Do you see it yet?

Neither do I. This is the box where my paradigm skills finally fail. I'm stumped. However, if someone does come up with a solution (please come find me) I can at least ask the next question, "can you do it with an irrational number of lines?" Going from positive, to zero, to negative, to rational, to irrational, is a meta pattern, a paradigm of paradigms. Its extension should be obvious, that is we can even ask the next question, despite not having answers to the previous two, “Can we do it with an imaginary number of lines?”

And that is the perfect segue.