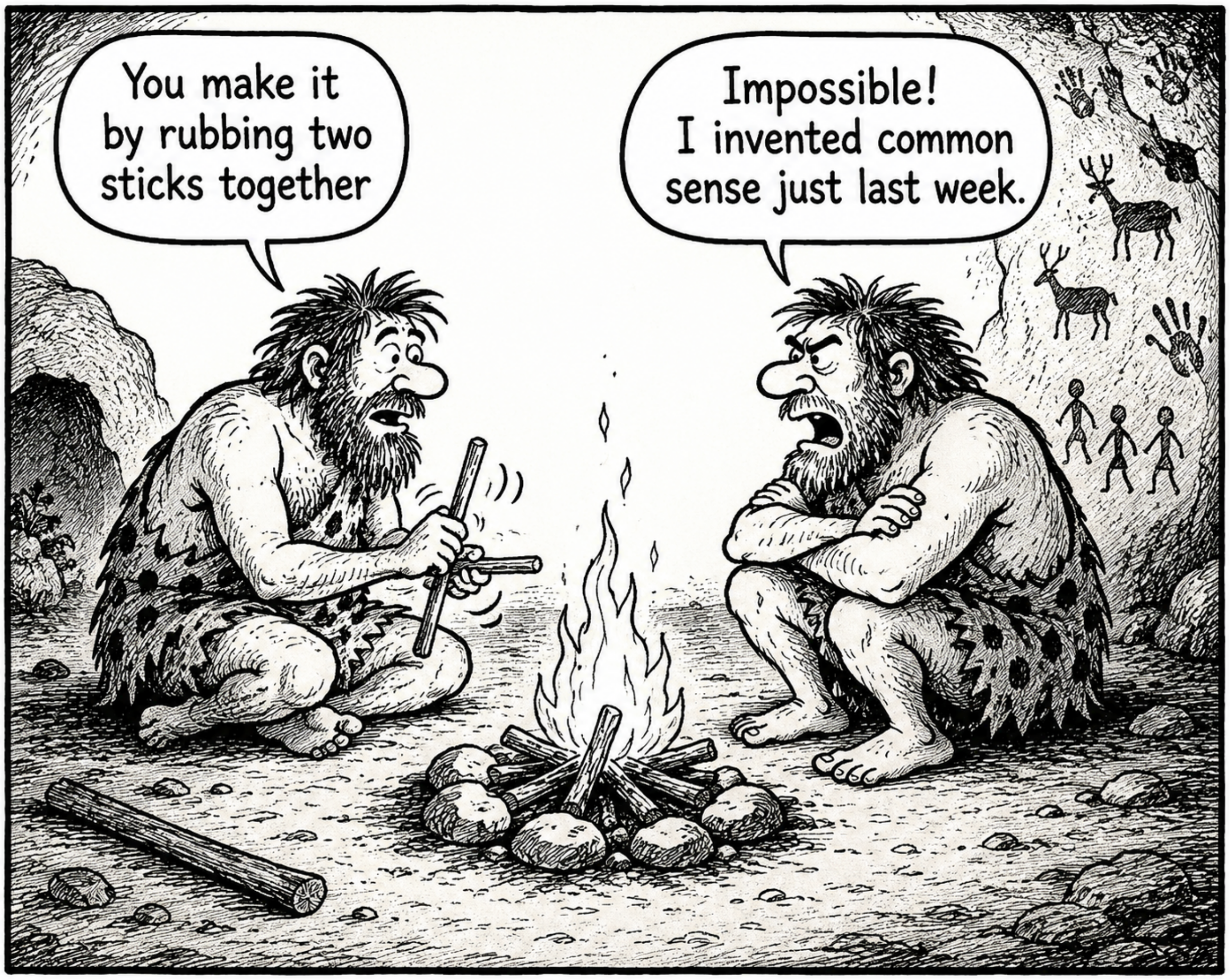
# Chapter 3

"Special Relativity"




*Commonsense*

Chapter 1 introduced the concept of paradigm, the background worldview that sharpens our perceptions in conventional situations but masks our hidden assumptions in unconventional situations. Chapter 2 introduced the unconventional situation of self-reference in logic, showing how it led to paradox (logical contradiction), where the conceptual challenge was inconsistency. The paradigm shift proposed there was to introduce imaginary truthvalues as the solution to logical paradox. It was further noted that self-reference is inherently nonlinear, thus multivalued, and that the new imaginary truthvalues, i & j formed a conjugate basis set with the classic Boolean truthvalues T & F.

In this chapter we consider the paradigm shift that led to the theory of Special Relativity. In everyday experience, the realm of common sense (what evolution has hardwired our brains to be efficient at), events occur on a vast stage of uniform space in a uniform progression of time. In this paradigm, space and time are independent of each other, and all observers agree on the numerical values of any event; its 3D spatial location (where it occurred), and its 1D temporal location (when it occurred). This commonsense view of reality was given scientific credibility by the invention of the theory of gravity and the rest of mechanics; and enhanced to mathematical exactitude by the invention of the calculus. This world view is now referred to as the classical (or sometimes Newtonian) world view. It is a powerful world view, an accomplishment that our species' takes deserved pride in having, finally, figured out. After some 12,000 years of recorded human history, it was only 400 years ago that this veil was lifted from our eyes. That it took so long to 'get it' is a bit of a surprise, almost an embarrassment.

As impressive as it is, as useful as it is, as accurate as it is at low speeds, it is not actually the way reality works. Like most paradigm shifts, this was a bit of a shock. Space and time are not the same for observers who are moving relative (there's the key word) to each other. They are not absolute, nor are they independent. Observers in relative motion will not agree on the location nor on the time a specific event occurs. The bigger the velocity difference, the bigger the numerical differences. Very weird. The modern term for this concept is *spacetime* and the speed of light is the thread that weaves space and time into a cohesive whole. Within limits, space can be converted into time, and time converted into space.

You can't make this stuff up. Reality is weirder than we can imagine.

There is an abundance of technical and lay material on special relativity. It seems wasteful to duplicate that information here, but the discussions to follow require a rather complete understanding of Einstein's most famous theory. Therefore, to avoid tasking the novice reader with more outside reading, or boring the more knowledgeable reader, a very concise summary of the theory of relativity will be presented. Think of this as a kind of engineering tradeoff between volume and completeness. (The Intrepid Reader may of course indulge themselves in any number of technical and lay books about relativity. We'll still be here next week.)

Below is a table listing the key concepts with correct formal jargon, along with a brief explanation of each concept. It's a little dry; read, skim, or skip at your preference. After the table are a few figures along with succinct summaries of the major concepts relevant to our purposes.

**Summary Table of Relativistic Concepts**

| Jargon | Concept |
|---|---|
| Event | A well localized observable for which both the spatial and temporal coordinates are known with an uncertainty low compared to other events of interest. |
| Frame of reference | A coordinate system capable of measuring both space and time, one that is in uniform motion, no acceleration. |
| Observer | The measurement taker, always associated with a frame of reference. |
| Spacetime | Refers to the tight coupling between space and time. |
| Minkowski space | A pseudo cartesian system on spacetime; where time is imaginary. |

| Jargon | Concept |
|---|---|
| Spacetime diagram | A typically 2D diagram of Minkowski space with time vertical and space horizontal; with time measured in seconds (s), and space measured in light-seconds (ls). |
| World line | The path of an object in Minkowski space. A straight line for uniform motion; vertical for an object at rest, at 45° for a photon. |
| Spacetime interval | The Minkowski interval between two events, $\Delta s^2 = \Delta x^2+\Delta y^2+\Delta z^2-\Delta t^2$. An invariant: all observers will compute the same interval between the same two events. |
| Lightlike interval | Spacetime interval of zero, the kind of interval light travels on. |
| Timelike interval | Connects two events in a timelike way, i.e., the earlier is a potential cause of the later. An object could travel from the earlier to the later at sublight speeds. Characterized by a numerical value less than zero. Conventional causality. Temporal ordering is a preserved quantity for timelike intervals, all observers agree on which event is first and which last, but they may not agree on which event is left and which right. |
| Spacelike interval | Connects two events in a spacelike way. To get from one event to the other requires faster than light (FTL) travel. Characterized by a numerical value greater than zero. Left/right ordering is a preserved quantity for spacelike intervals, all observers agree on which event is to the left and which is to the right, they may not agree on which event is earlier and which later. |
| Light cones | The set of world lines a source of light makes. In a 3D Minkowski space takes the shape of an actual cone. Separates timelike regions from spacelike regions. |
| Doppler shift | The shift in the frequency of light due to relative motion between the source and the detector. Slightly different formula than for waves in a medium, significant only at high relative velocities. |
| Red shift | Source and detector are moving away from each other. The light waves are stretched out to a lower frequency. For visible light, the shift is toward the red end of the spectrum. |
| Blue shift | Source and detector are moving toward each other. The light waves are compressed to a higher frequency. For visible light, the shift is toward the blue end of the spectrum. |
| Length contraction | The effect where objects in two frames of reference in rapid relative motion appear shorter to an observer in the other frame. |
| Time dilation | The effect where time in two frames of reference in rapid relative motion appear slower to an observer in the other frame. |

| Jargon | Concept |
|---|---|
| Relativistic mass | The effect where mass in two frames of reference in rapid relative motion appear to increase to an observer in the other frame. |
| Simultaneity | The concept of two events happening at the same time. Not an invariant in relativity. |
| Lorentz Transform | The mathematical equation that transforms the spacetime coordinates from one reference frame to another. |
| Timelike cause (TC) | Conventional causality. Two events connected by a timelike interval where the earlier event is the cause and the later event the effect. The cause is in the past light cone of the effect. |
| Common timelike cause (CTC) | An event (cause) in both past light cones of two future events (effects). The two future events may be spacelike separated, in which case timelike causality cannot explain correlations between the two effect events. |
| Spacelike cause (SC) | A hypothetical cause between two events which are spacelike separated. Implies FTL travel or communication. |
| Common spacelike cause (CSC) | A hypothetical cause where one event outside the light cones of two other events (the effect events) explains the correlations between them. The effect events may be either timelike or spacelike separated. |
| Symmetry | In relativity physics, space and time are very nearly symmetric, not so in Newtonian physics. |

**Visualization**

The easiest way to understand relativity is via spacetime diagrams. A coordinate system is chosen to align with the velocity vector between two observers thus reducing a 4D system to a 2D one: one dimension of time (vertical) and one dimension of space (horizontal). Much easier to visualize, much easier to draw on a whiteboard, or sketch on a piece of paper, or present as a figure in a book. An arbitrary origin is chosen from which two light cones are drawn: one upward in time and one downward. These are referred to as the future and past light cones respectively. The reason for the name 'light cone' is that if a 3D spacetime diagram is used the world lines for light rays form a cone. On the 2D 'lite' version (sorry) the world lines for light rays are just a pair of diagonal lines; +45° and -45°.

The next figure shows three spacetime diagrams; the first shows the future and past light cones (dotted lines) with the straight vertical world line of the origin (solid line), the second shows the world line of an object moving to the right, and the third adds the world line of an even faster object moving to the left. In this third spacetime diagram, both moving objects arrive at their respective destinations at the same time (dashed line). The world lines of the light cone are *lightlike* intervals, the world lines of the two objects in motion are *timelike*, as is the world line of the stationary origin. The line connecting the two arrival events is the line of simultaneity; it is a *spacelike* interval.

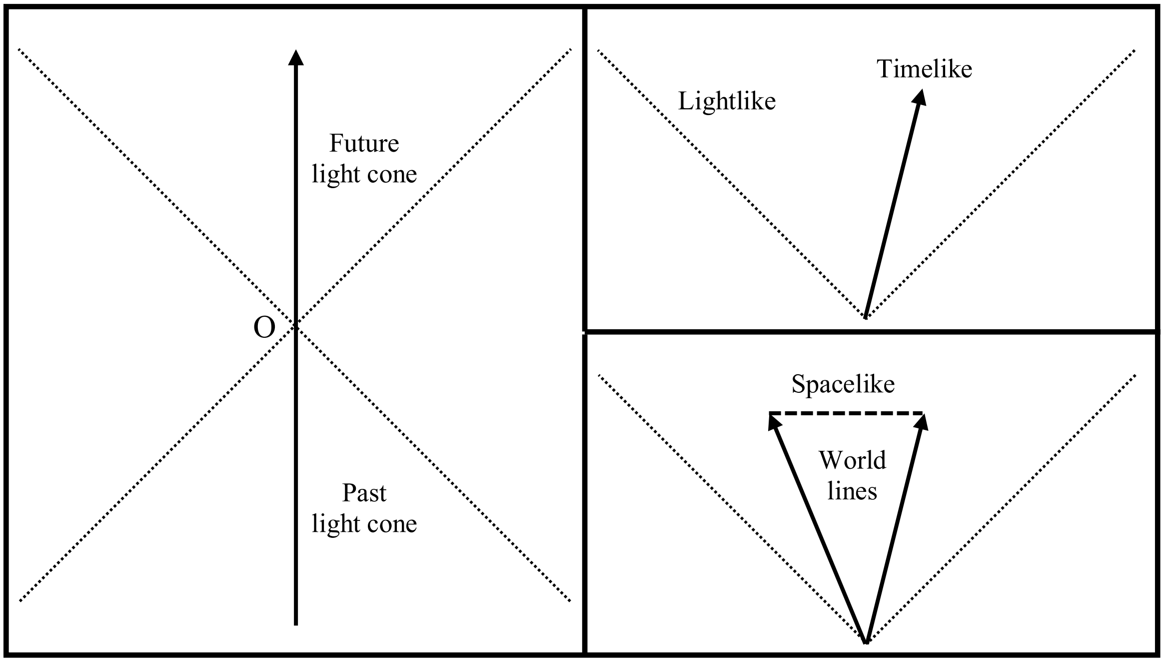



Figure 1 – **2D Spacetime Diagrams**: The horizontal is space (units of light-seconds, ls), the vertical axis is time (units of seconds, s). In this system of units, the speed of light (c) is 1 ls/s, thus photons travel along worlds lines at +/- 45° angles. The left panel shows past and future light cones which connect at the origin. The upper right panel shows the world line a fast moving object traveling to the right. (An object at rest has a vertical world line.) The lower right panel two fast moving objects traveling in opposite directions. In one and only one frame of reference the detection of these two objects will be seen as simultaneous, which implies a spacelike interval between them that is horizontal.

Note that both objects have world lines at less than a 45° angle (measured from the vertical), they are moving at sublight speeds. An object moving faster than light would have a world line more horizontal than vertical, a spacelike interval.

In these diagrams, time is measured in seconds (s) and distance in light-seconds (ls). Thus, velocity is measured in light-seconds per seconds (ls/s) and is always between plus one and minus one.

This should be relatively clear (ahem…), we have a flexible diagram motif, concise jargon, and an integrated nearly symmetric set of concepts. Now we add the complexity (sit up Intrepid Reader) this is the good stuff. What does that last spacetime diagram look like to an observer in constant relative motion in the horizontal direction?

The next figure shows two spacetime diagrams for the same physical system as above, but from the frame of reference of two new observers; the first shows an observer moving to the left, the second shows an observer moving to the right. We'll let them have the same speeds, just in opposite directions. Note, that the lightlike intervals are unchanged, but the timelike and spacelike intervals have different slopes and different lengths. These are called Poincaré rotations.

A couple of notes about the timelike intervals, the world lines of the objects in motion; first they have been color coded to reflect Doppler shifts, second their axial projections are different.

When a light source and an observer are moving towards each other, the light will be blue-shifted, higher frequency (shorter wavelength). Symmetrically, when a source and an observer are moving away from each other, the light will be red-shifted, lower frequency (longer wavelength).

While the intervals all have the same length in Minkowski space (the time coordinate is treated as imaginary), their axial projections are different. Minkowski lengths are invariants, they are not observer dependent, all observers will calculate the same numerical values. Physicists love their invariants. But the axial projections will be different; timelike intervals will be taller (time dilation), and spacelike intervals will be shorter (length contraction). Space has been converted into time, and time has been converted into space. Again, you can't make this stuff up.

Note the violation of simultaneity; the left moving observer sees the left moving object arrive first, while the right moving observer sees the right object arrive first. Simultaneity is not a preserved quantity in relativity – it is not an invariant.

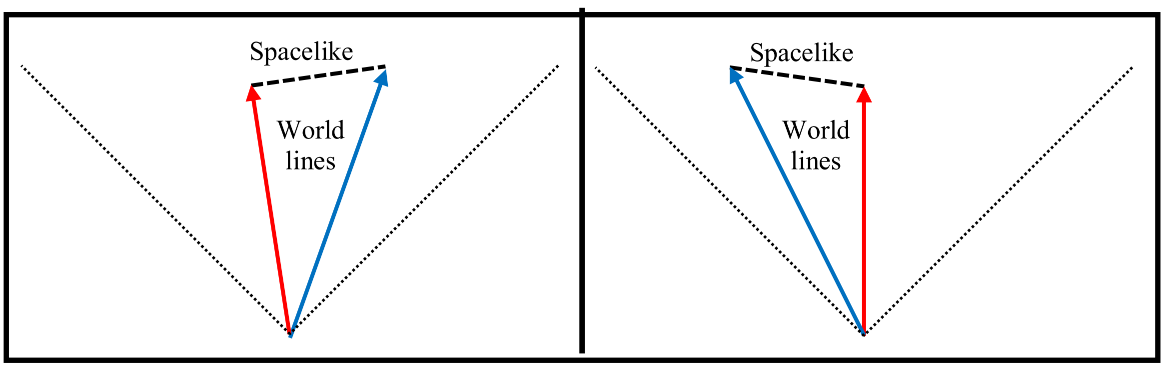



Figure 2 – **Poincaré Rotations**: Left panel, observer moving to the left; right panel, observer moving to the right. While the light cones don't change, the timelike and spacelike intervals do. Neither observer sees detections of these two objects as simultaneous. However, the 'length' of the spacelike interval (measured in Minkowski space, where the time dimension is imaginary) is an invariant. The object world lines have been color coded in accordance with their Doppler shifts (red or blue shifted).

The world lines of timelike intervals always grow in an upward direction; causality is from the past to the future. A classical time machine would have a world line that grew in a downward direction, still timelike, but backwards. Our universe does not appear to support backwards in time causality. Even in spacetime, space and time are not quite perfectly symmetric. Is this weird or not?

The question is the in-between world lines, the spacelike intervals, those more horizontal than vertical lines. If spacelike causality exists in our universe, it will occur along spacelike intervals, but that would imply faster than light (FTL) effects. However, observers will not agree on whether the causality is merely super luminal, or instantaneous, or even traveling into the past. Past and future are not preserved quantities for spacelike intervals – effects may precede their causes. But there is a symmetry here. Timelike intervals preserve temporal ordering, but not spatial ordering, while spacelike intervals preserve spatial ordering, but not temporal ordering. Beautiful.

This symmetry can be made a little more complete by noting that as timelike causality implies so called common cause, timelike of course, then if spacelike causality exists, it also implies a common cause, call it common spacelike cause. The next figure shows these four types of causality.

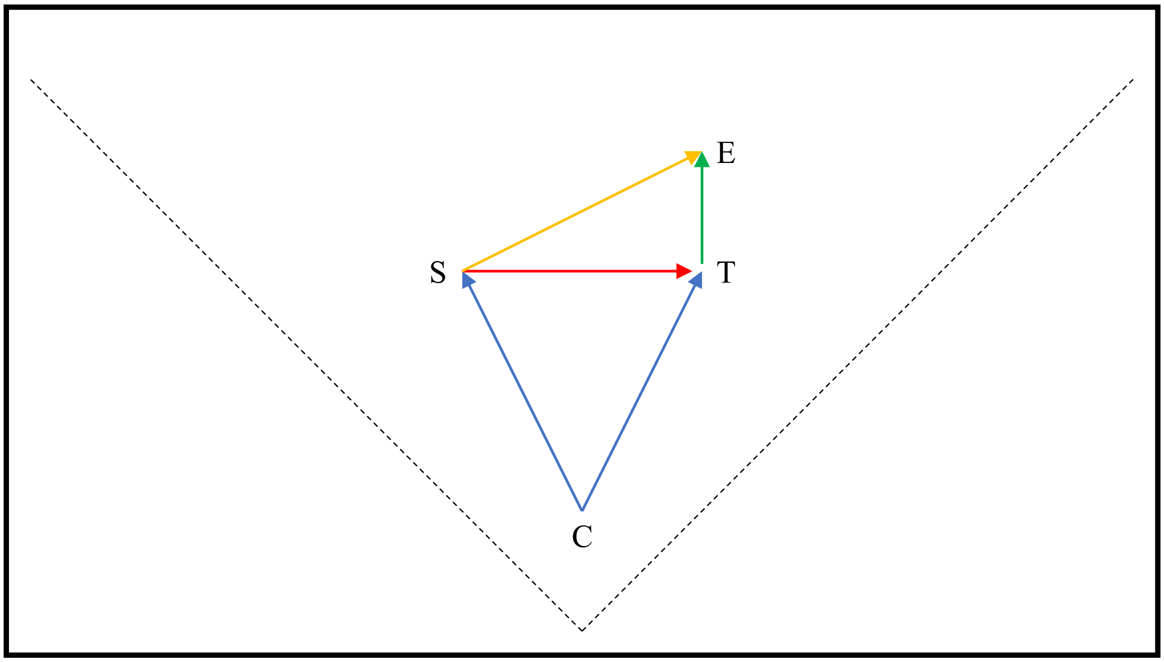



Figure 3 – **Four Types of Causality**:
T could be the timelike cause of E (TC).
C could be the common timelike cause of both S & T (CTC).
S could be the spacelike cause of T (SC).
S could be the common spacelike cause of both T & E (CSC).

But there is also an asymmetry. Time is *not* isotropic, so causality is always from past to future, but space *is* isotropic, so there is no way, apparently, to determine the direction of causality. Without that, observers cannot agree on the physics; causality is broken. Thus, Dogma would assert, spacelike causality is *verboten*. QED.

Paradigm of course, wouldn't leave it at that, "But what if we could, what if different observers could agree, on a case-by-case basis, on which event was the cause, the left one or the right one? The physics would be relativistically consistent; everyone would agree on which event was the cause and which the effect. Causality restored."

So, give Paradigm a little rope; let's assume, for the moment, that relativistically consistent spacelike causality exists for some systems.

And therein lies the potential and the peril; two spacelike causes, with opposite slopes can be used to implement backwards in time causality. Now we have the specter of closed causal loops in spacetime, self-reference, once again, intrudes into our carefully constructed view of reality.

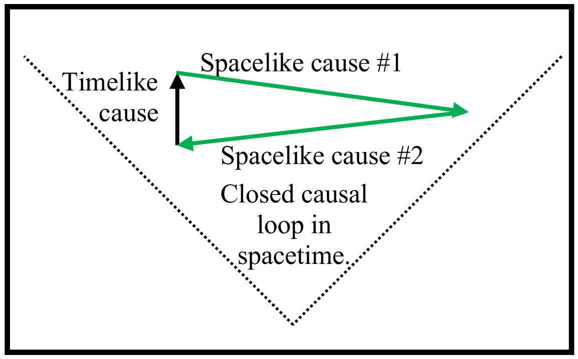



Figure 4 – **A Closed Causal Loop in Spacetime**: Two spacelike causes, with opposing Poincaré rotations, can be used to create a closed causal loop in spacetime. Surely physics cannot tolerate such a construct, for what is to prevent the causal loop from being paradoxical, a cause that denies itself? The answer is in the question: look for the '*what*,' for the thing that prevents paradoxical loops, but otherwise allows them.

Introducing self-reference into spacetime creates all the usual problems; paradox and indeterminacy, contradiction and acausality, confusion and dogmatic reflexes. Spacelike causality cannot be allowed; it isn't relativistically consistent and if it was, it would allow causality contradictions. God gives up, "You got me" and the universe dissolves back into the dark chaos from which it came.

The conundrum is that quantum mechanics seems to imply spacelike causality – something is wrong, either relativity or quantum mechanics. Reality, stubbornly, is still here.

We are in a box.