

Chapter 4

“Quantum Mechanics”



Amber Particles of Grain

If relativity was a shock, quantum physics was a stupefying assault on reason itself. There is nothing in everyday experience that can be related to it. It violates common sense in almost every way possible. The quantum realm continues to deny our intuition any foothold; all of our mental models are flawed, fundamentally incomplete. Even Einstein could not reconcile himself to this description of reality as being anything other than a temporary place holder until a finished theory could be devised. Despite being our most successful physical theory (it has been verified in some situations to an astonishing 16 decimal places), it is the only modern theory plagued by the problem of multiple interpretations. Depending on the taxonomy chosen, there are some 6 to 9 different schools of interpretation, different ideas of what quantum mechanics *means*, none of which make any different experimental predictions with which to decide between them. It's not even clear if

any of the ones on the list are correct. It's not just that we don't know which ones are wrong, there's no guarantee they aren't all wrong.

In contrast, while the Copernican revolution, the realization that the earth revolves around the sun, rather than the sun revolves around the earth, is nonobvious, it is still possible to create a model of both the earth centered system and the sun centered system with everyday objects. Even though the experiential perception goes against it, one can visualize the two competing systems, both mentally and through physical models.

Not so for quantum physics.

The conundrum is even more remarkable given that quantum theory has been more rigorously tested, in a broader range of phenomenon, energy, size, scale, precision, and diversity than any theory, ever. No contradictions to it have ever been found, none are expected. We have been forced to it, kicking and screaming. We've been unable to find any other mathematical formulation that is able to correctly predict experimental outcomes; but the math makes no sense. If the purpose of science is to *understand* reality, then quantum mechanics thumbs its nose at us.

One pithy wit summed up the situation with this sarcastic quip, "Shut up and compute." Our embarrassing inability to explain quantum physics has led many scientists to a kind of professional resignation, abandoning the mission of ever being able to figure out what it means. In defeat, they have retreated to repositioning the objective of science to be one of prediction, rather than one of understanding. Those less frustrated simply recognize that there is a practical challenge to deal with; in order to have a successful career one must focus on prediction, and hope that at some future date a breakthrough will occur that can restore understanding to its central position as the prime motivator of the scientific endeavor.

As there was for relativity, there are many good books (many, many more in fact) that describe quantum mechanics; they range from the scholastic near textbooks, to books for the intelligent layman, down even to the rather humorous "Quantum Mechanics for Babies." There is no shortage of attempts to make the counter-intuitive nature of reality at very small scales as accessible to the public as possible. It is not unlike the temptation science fiction writers have to the lure of the time travel story; the science fact writer finds the intellectual challenge of describing quantum mechanics just as irresistible.

For the work to come, the most important concept will be that of entanglement, a quantum phenomenon that has absolutely no classical analog. Entanglement is where two or more particles that have interacted in the past maintain a kind of connection that does not diminish with distance. It is at odds with relativity. To understand entanglement, or perhaps more modestly just to properly characterize it, requires familiarity with a handful of concepts that make up this fascinating field called quantum mechanics. It's a big elephant, best eaten one bite at a time. Therefore, we shall take our time. Each of the next few chapters build upon the previous. With each bite we will be extending a conceptual framework upon which to hang all the fundamental concepts of quantum mechanics that lead, inevitably, to the phenomena of entanglement.

Consider the rest of this chapter course one in a four-course meal.

Discrete versus Continuous

In classical physics, the various traits that an object might have, position, velocity, energy, etc., were envisioned as a continuum of possible values. In quantum physics, this common-sense notion is violated. Quantum systems in general can only take on discrete values; think dashed line rather than number line. This ‘quantization’ of value is where quantum physics gets its name.

Depending on the trait (and on the quantum system being investigated), the dashes may be very fine, almost indistinguishable from a continuum, to extremely coarse. Since uncovering this unexpected behavior took time and was subject to the vagaries of history, accident, timing and personality, some concepts go under the guise of different names; one name more illustrative when the dashes are fine, a different name more communicative when the dashes are coarse.

There are times when the precision and conciseness of mathematical notation simply leaves prose behind. This is one of them. The jargon is standard to the field and will be introduced step-by-step, but make no mistake, your pedagogical author is asking you, Intrepid Reader, to invest in mastering the notation.

State

The starting point for any investigation is, “What are the objects and what are the values of their various properties?” In general, one is interested in only some of the properties, the rest are acknowledged but ignored. The term for the set of values of interest is *state*. A driver might ask, “what is the state of the car?” and by that they would like to know the location and speed. A repair technician might ask the same question but would be interested in how thick the brake pads and how deep the tire tread. The dealer would be interested in the trim and mileage. You get the idea.

The concept of state applies to individual objects, but also to a collection of individual objects. The latter is typically referred to as the *system*. For quantum particles, an example of typical traits would be polarization for photons and spin for electrons. Note the inherent ambiguity and redundancy of language here. The things of interest might be called *objects* or *particles*, the attributes might be called *properties* or *traits*. The state might refer to an *individual* thing or to the whole *collection* of things. This is why formal notation is important; it is designed to remove the ambiguity and redundancy of conventional prose.

You, Intrepid Reader, are now ready for your first quantum equation.

The system state will be represented as an equation. The single term on the left represents the state of the entire system, the terms on the right are possible states of the particles in the system. The overall system state is represented by the capital Greek letter Ψ (psi), typically used to indicate the wave function. The potential values of the particles will be indicated by a letter inside a Dirac ket, $|x\rangle$, an asymmetric bracket notation where the letter inside indicates the value of the trait. Thus, a single horizontally polarized photon would be represented like this

$$\Psi = |H\rangle \quad (1)$$

The choice of symbol inside the ket is generally arbitrary, typically chosen for reasons of clarity. In this case the H is intended to suggest ‘horizontal’.

If there were two particles in the system, and the second one had vertical polarization (V), then the state of the system would be represented like this

$$\Psi_{12} = |H\rangle_1|V\rangle_2 \quad (2)$$

The addition of subscripts reveals the mapping between the trait values and the particles; typically, a single alphanumeric character. In this case, particle 1 is horizontally polarized and particle 2 is vertically polarized.

Superposition

You might think that a particle must be either horizontally polarized or vertically polarized. While the common sense view, the word *or* is a paradigm assumption. In quantum mechanics, the word *and* is allowed, indeed unavoidable. This is the principle of superposition; a particle can be in an indefinite state of polarization between two possibilities. Competing ways of breaking the *or* paradigm are to use the word ‘both’ or even the word ‘neither’; the particle is both horizontally polarized and vertically polarized, or the particle is neither horizontally polarized nor vertically polarized.

Do you see the problem with trying to use prose to communicate this concept? Instead, we’ll use the notational precision of mathematics to disambiguate, and the prose word for the concept will be simply *superposition*. Here is the quantum mechanics’ equation for a single particle in a superposition of polarization states

$$\Psi_1 = \{\alpha|H\rangle_1 + \beta|V\rangle_1\} \quad (3)$$

The Greek letters α and β are called *weights*. They are complex numbers, so they encode both size and phase. The term *complex* as used here is a taxonomical category, a type of number to be distinguished from other types of numbers; it is not intended to imply complicated. A complex number has two parts, one part is called real, the other is called imaginary.

The weights specify the subtleties of the superposition, in part how much horizontal versus how much vertical. From them it is possible to compute the odds of what value of polarization will be observed when this particle’s polarization is measured. To make this calculation easier, a constraint on α and β is enforced; the sum of their squares must be unity. Thus

$$\alpha^2 + \beta^2 = 1 \quad (4)$$

Now the square of each weight is just the probability that its associated polarization will be observed. When this constraint is followed, the weights are said to be *normalized*.

While the use of weights is formally correct, in many situations they can be almost eliminated. When the odds of each polarization are equal and the phase angle between them is either 0 or 180 degrees (remember, they are complex numbers; think polar coordinates) then the equation of superposition can be cast in this simpler form.

$$\Psi_1 = \frac{1}{\sqrt{2}}\{|H\rangle_1 \pm |V\rangle_1\} \quad (5)$$

The equal probability of the two polarizations is now indicated by the radical, and the two possible relative phases by either the plus or minus sign.

If this system is measured, then the particle (particle 1 is the only particle in this system) has a 50% chance of being horizontally polarized and a 50% chance of being vertically polarized. For those situations where this notational simplification is appropriate, each term on the right-hand side of the equation has an equal probability of being the outcome of a measurement. Each measurement selects a subset of terms, typically half of them, always randomly. The value under the radical sign is just the number of terms. Remember that the mapping from weights to probabilities is to square the weights; thus squaring $\frac{1}{\sqrt{2}}$ is just $\frac{1}{2}$. Each outcome has a 50/50 chance.

A particle in states like this is said to be in a maximal superposition; the probabilities have equal magnitude (1/2). When the weights are not of equal magnitude, then one polarization is more likely than the other. In the extreme, when one weight is zero, then the particle is 100% in the other state, equivalent to a classical state.

In this form, it is much easier to introduce the next concepts, the ones alluded to in the Paradigm Discourse; how hard is half white and half black, and soft is half black and half white; and vice versa.

Measurement Bases

Horizontal and vertical polarizations are not the only possible polarizations. There are others, and there is a tight and specific mathematical relationship between all of them. At the heart of this relationship is the concept of a measurement basis. Horizontal and vertical are one basis; here is another, diagonal and slant. For short, call the first one the *grid* basis, and call the later the *angled* basis. A photon in an equally weighted superposition between diagonal polarization and slant polarization would be represented in the angled basis like this

$$\Psi_1 = \frac{1}{\sqrt{2}}\{|D\rangle_1 \pm |S\rangle_1\} \quad (6)$$

For the perfectionists out there, baseball players run the bases (pronounced base-es), chemists use acids and bases (also pronounced base-es), but physicists use bases (pronounced base-ezes). Go figure.

And now, drum roll, we consider the relationship between the grid and the angled bases.

Conjugate Bases

The trait values in one basis have a relationship with the trait values in a basis conjugate to it. The grid and angled bases are conjugates of each other. Here are the four relationships between the pure states

$$\begin{aligned}
 |D\rangle &= \frac{1}{\sqrt{2}} \{|H\rangle + |V\rangle\} \\
 |S\rangle &= \frac{1}{\sqrt{2}} \{|H\rangle - |V\rangle\} \\
 |H\rangle &= \frac{1}{\sqrt{2}} \{|D\rangle + |S\rangle\} \\
 |V\rangle &= \frac{1}{\sqrt{2}} \{|D\rangle - |S\rangle\}
 \end{aligned} \tag{7}$$

The symmetry is obvious. Note how a pure state in any one basis is in an equally weighted maximal superposition in the conjugate basis. A measurement in a basis in which the quantum object was in a superposition will collapse to one of the possible trait values. The term *collapse* here has a very specific physical meaning; it means that the very act of measuring the object will *change the value* of its property. Classical systems do not behave this way.

However, successive measurements in the same basis will *not* change that value but having collapsed to a pure value in one basis, the quantum object is now in a superposition in the conjugate basis: an equally weighted, maximal ambiguity in the conjugate basis. This is the behavior exhibited by Reason's uncertainty box. Simply replace H & V above with black and white, and D & S with hard and soft.

The very definition of horizontal is half diagonal, half slant ($|D\rangle + |S\rangle$), the very definition of vertical is half slant, half diagonal ($|D\rangle - |S\rangle$). To be verbose, the very definition of diagonal is half horizontal, half vertical ($|H\rangle + |V\rangle$), and the very definition of slant is half vertical, half horizontal ($|H\rangle - |V\rangle$).

How cool is that.

But you ask, "Now wait, they are still all polarizations. Dogma's objection was that *firmness* and *color* are different things, measured differently, measured in different units. Hardness and softness are conceptually different than blackness and whiteness."

Time for the next chapter.