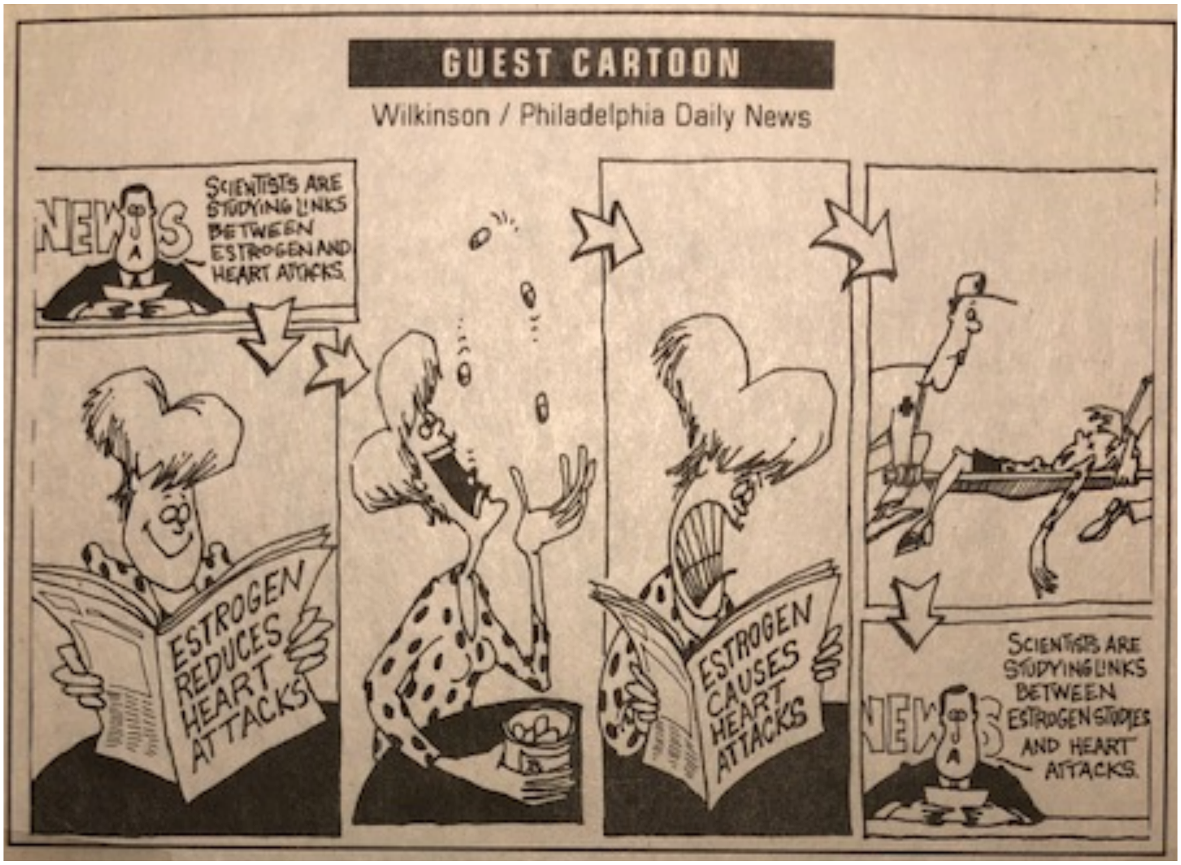
Chapter 5

"Uncertainty"



Credit – Wilkinson: "Estrogen Studies" (need permission)

At the center of the mystery of quantum mechanics is probably the idea of superposition. Indeed, Feynman, with his usual succinct wit, called it the only mystery. He implied that if we could only get our heads around this one, the rest might all fall into place. First consider the word itself; super-position. It literally means multiple positions. At the quantum level it is possible for an object to be in two places at once.

What?

We never experience superposition, trapped as we are at the classical level. A thing is either here *or* there, not here *and* there. What would it feel like if you were in a superposition, in two places at once? Not a copy, not a duplicate, not a clone; the real you, both here and there. If you stubbed your toe, would it happen in both places? Coughed? Thought? Made a decision? Is there any way at all to make sense of this concept at the classical level, to align it with common sense? (Spoiler alert, it's pages and pages away.)

But the phenomenon of superposition is more general than just multiple positions. Superposition (as a bit of technical jargon), means more than just being in two (or more) *places* at once; a quantum object can simultaneously have multiple values of nearly any physical quantity, not just position. The list is long; momentum, energy, time, spin, polarization, etc. etc. etc. It gets weirder. The physical quantities tend to come in pairs which are deeply coupled with each other. The pairs may have the same units (such as spin) or different units such as position and momentum. In either case, a basis must be selected in order to perform a measurement. The result of that measurement may be a mono value in the selected basis, but in all bases (last time, pronounced base-ezes) conjugate to the selected basis the quantum object will be in a maximal superposition.

Thus, for an object like an electron that can have only half-integral spin, if its spin is +1/2 along the z-axis, it will be in a superposition of +1/2 and -1/2 in both the x and y-axis. (Yep, there can be more than one conjugate basis.) If a position measurement is accurate enough, that is if the position is now known with small uncertainty, then the momentum will have a large uncertainty, and thus be in a superposition of many momentum values. That position and momentum are conjugate bases came as quit a surprise. Dogma's assertion that they are different concepts, defined differently, measured differently and measured in different units, should have been enough for them to be uncoupled (orthogonal) concepts. At the classical level they can be treated as such without loss, but at the quantum level, they are aspects of the same thing. Reason's 'uncertainty' box is an analogy for superposition based on conjugate bases. Like spin, there are only two values, but like position and momentum, firmness and color are measured in different ways.

It is conjugate bases that provide the real foundation for the uncertainty principle. While the idea that measuring a system disturbs it is sound, and very applicable in the classical realm, unlike the classical realm the amount of disturbance cannot be reduced to arbitrarily low levels by experimental technique. Nature places her own limits.

Let's dig a little deeper.

Action

Units are the foundation for how to measure things quantitatively. There are basic units and there are combined units. The basic units are for length, mass, and time. In the metric system, quantities of those types are measured in meters (m), kilograms (Kg) and seconds (s), respectively.

Examples of combined units typical of both classical and quantum physics are velocity (m/s), momentum (Kg-m/s), energy (Kg-m^2/s^2), etc. The combined unit most central to quantum physics is *action* (Kg-m^2/s). It is the unit of Planck's constant

$$\text{h} = 6.67\ x\ 10^{-34}\ Kg \cdot m^2/s \tag{1}$$

A closely related constant, called h-bar, is the unit of spin. It is just Planck's constant divided by 2π

$$\hbar = \frac{h}{2\pi} = 1.061\ x\ 10^{-34}\ Kg \cdot m^2/s \tag{2}$$

In some senses, h-bar is the more fundamental constant, but due to the accidents of history, the peculiar path of discovery and inspired insight that eventually supplanted the paradigm of classical physics with the paradigm of quantum physics, the first version gets the formal name and the status as a universal constant.

The Uncertainty Principle

Uncertainty in a physical quantity is generally indicated with the Greek letter delta, 'Δ'. We now have the mathematical tools to formally specify the uncertainty principle. What we do is take the combined unit action and divide it up into two parts. (Paradigm is wondering if we've ever tried dividing it up into three parts.) There are several ways to do so. At this point in the development, we'll only consider the two best known examples.

Position-Momentum

First let's split position out of action; that leaves momentum. Position and momentum are in an uncertainty relationship. The combined product of their uncertainty can never be less than h-bar

$$\Delta x \Delta p \geq \hbar \quad (m)(Kg \cdot m/s) \tag{3}$$

The two variables represent position (x) and momentum (p). Unlike Reason's uncertainty box, position and momentum can have more than two possible values, but like Reason's uncertainty box, they are different types of things. Dogma is quite correct, they are defined differently, and they are measured differently, in different units; position in meters, momentum in kilogram-meters/second. If we measure position perfectly, the momentum becomes maximally uncertain; and in perfect symmetry, if we measure momentum perfectly, the position becomes maximally uncertain.

Position and momentum are conjugate bases. A position value is a superposition of momentum states, and a momentum value is a superposition of position states. It is perplexing for it seems as if position and momentum are each made out of the other.

Time-Energy

Next, let's split time out of action; that leaves energy. Time and energy are in an uncertainty relationship. The combined product of their uncertainty can never be less than h-bar

$$\Delta t \Delta E \geq \hbar \quad (s)(Kg \cdot m^2/s^2) \tag{4}$$

The two variables represent time (t) and energy (E). They are another example of Reason's uncertainty box and of Dogma's complaint. Measuring either one too well forces great uncertainty into the other.

Time and energy are conjugate bases. A time value is a superposition of energy states, and an energy value is a superposition of time states. It is perplexing for it seems as if time and energy are each made out of the other.

Red Herrings

A red herring is a type of paradigm puzzle where the perpetrator has included more information than strictly required for solution, and perniciously, with malice aforethought, has carefully chosen the unneeded information so as to trick our brains into attacking the puzzle from a perspective from which there is no solution. The trick to solving a red herring puzzle is in being able to correctly choose what to ignore.

As Paradigm's story unfolds, be on the lookout for situations where nature has (or maybe just scientists have) provided more information than strictly necessary. Too much information can be a box.