Chapter 7
"Entanglement"

Plagiarism

Superposition violated our expectations on the *particular* values that an individual object can take on, specifically, too many. Entanglement violates our expectations on the *sets* of values that a group of objects can take on, specifically, too few.

The first hints of this came about in the course of the famous series of debates between Einstein and Bohr in the 1930's, when Einstein, Podolsky & Rosen published a paper arguing that quantum mechanics was incomplete. The paper proposed an experiment intended to demonstrate that a way around the uncertainty principle existed. While Bohr (after a sleepless night) was able to argue that the conclusion was wrong, the thought experiment itself was formally correct. In it two particles interact (or are created) in such a way that their respective values are no longer independent. They have been entangled. Such states are now referred to as EPR[1] states and particles so entangled maintain a correlation that does not diminish with distance.

It is truly spooky-action at a distance, and Einstein never reconciled himself to this property of quantum mechanics. He couldn't argue against the math but regarded this outcome as evidence that quantum mechanics was not yet a complete model of quantum physics.

We now have the mathematical tools to take a look at these group states with too few states.

Separable States

Consider a quantum system consisting of two particles with only two possible values for some trait. The classical view can be modeled by flipping two coins. Each coin has a 50/50 chance of coming up heads or tails, independent of each other. In the quantum realm, such a situation would be represented this way

$$\Psi_{12} = \frac{1}{\sqrt{4}}\{|H\rangle_1|H\rangle_2 \pm |H\rangle_1|T\rangle_2 \pm |T\rangle_1|H\rangle_2 \pm |T\rangle_1|T\rangle_2\} \tag{1}$$

Each of the four possible outcomes have an equal chance of happening. The value of the first coin provides no predictive value for the value of the second, nor vice-a-versa. The two objects are not correlated. They are said to be in *separable* states. Why, because their states can also be represented as two equations, this way;

$$\begin{aligned} \Psi_1 &= \frac{1}{\sqrt{2}}\{|H\rangle_1 \pm |T\rangle_1\} \\ \Psi_2 &= \frac{1}{\sqrt{2}}\{|H\rangle_2 \pm |T\rangle_2\} \end{aligned} \tag{2}$$

Just multiply them together to get equation (1).

Entangled States

In quantum systems, however, it is possible to have states which are not separable. To make this more evident, here is equation (1) presented in a more 2D way

$$\Psi_{12} = \frac{1}{\sqrt{4}}\begin{Bmatrix} \pm|H\rangle_1|H\rangle_2 \pm |H\rangle_1|T\rangle_2 \\ \pm|T\rangle_1|H\rangle_2 \pm |T\rangle_1|T\rangle_2 \end{Bmatrix} \tag{3}$$

The four possibilities now form a table of two rows and two columns. Along the prime diagonal, the trait values of the two quantum objects are correlated, along the cross diagonal they are anti-correlated. This later case corresponds to the spooky-coins in the Discourse. The state of those two coins were anti-correlated, and would be represented like this

[1] (*Ahh*; Experiment, Proof & Reason)

$$\Psi_{12} = \frac{1}{\sqrt{2}}\left\{ \begin{matrix} & \pm\, |H\rangle_1 |T\rangle_2 \\ \pm|T\rangle_1 |H\rangle_2 & \end{matrix} \right\} \tag{4}$$

If one coin comes up heads (particle is horizontally polarized), the other one comes up tails (vertically polarized). If one spooky-coin was gold, the other will be silver. There is no way, mathematically, to provide a definitive value for either object's trait value.

For correlated states, this would be the representation

$$\Psi_{12} = \frac{1}{\sqrt{2}}\left\{ \begin{matrix} \pm|H\rangle_1 |H\rangle_2 & \\ & \pm\, |T_1 |T\rangle_2 \end{matrix} \right\} \tag{5}$$

If one coin comes up heads (particle is horizontally polarized), so will the other one; and similarly, if one coin comes up tails (particle is vertically polarized), so will the other one. If one spooky-coin was gold, the other will be too; if silver, then so also the other.

When states are non-separable, they are *entangled*. The definitions are mutual and exclusive. Neither particle has a well-defined trait value. It's properties now depend on another. It is in this sense that an entangled quantum object ceases to have an independent reality.

Conflict with Relativity

The implications of entanglement are, to say the least, perplexing. The equations above, the mathematical representations of non-separable states, are independent of distance; the entanglement does not get weaker with increasing separation. No matter how far apart the particles are, they remain perfectly entangled until measured. Then, apparently upon a measurement, both particles immediately take on a discrete trait value. Their quantum states collapse from a superposition state to a pure state – instantly. The problem; instantaneous is not a relativistic invariant.

If the two particles have separated far enough that each can be measured in a period of time less than it would take to send a signal from one to the other at the speed of light, then they have become spacelike separated. If measuring one particle of an entangled pair causes both particles to assume a pure state, then we are looking at a spacelike cause. Suddenly it appears possible to send a superluminal signal, and that violates the precepts of relativity.

Somehow, our two most successful physical theories have come into conflict with each other.

However, in order to send a signal both particles must be measured; so which measurement event causes the collapse? It can't be the earlier one, for the temporal ordering of spacelike separated events is not a relativistic invariant. Some observers will see the event on the right as the cause, others as the event on the left, and one special observer will see the two measurements as occurring at exactly the same time. Which event is the cause? Which is the effect?

Surez & Scarini

This was so troubling that two researches (Surez & Scarini) decided it should be codified into a falsifiable hypothesis. They proposed that one event really was the cause, that the two measurements shared a common frame of reference and that within this shared frame, one of the measurements, no matter how tiny the time difference, had to precede the other. They argued that the frame of reference of the non-measuring observers, those who saw one event as earlier than the other was irrelevant; the shared frame of reference of the measurement events was a kind of situation dependent absolute frame, and the only one that mattered physically.

Further they devised an ingenious experiment to test this idea; force the two measurement events to be in different frames of reference. Brilliant. An EPR pair of photons was to be generated and sent in opposite directions. Each would be subject to a measurement event, the two events being spacelike separated from each other, but with a twist; each measurement event would be in rapid relative motion with respect to the source of the EPR pair, and opposite to each other. In the relativistic reference frame of each measurement, the local event would be earlier in time, i.e., each event was, from their point of view, earlier in time than the other. Thus, there was no longer an unambiguous temporal ordering; no situation dependent absolute frame. Their prediction, or more accurately, their falsifiability criteria, was that the quantum correlations would disappear. The experiment was carried out[i]. The result? The correlations *did not* disappear. Their hypothesis was falsified. Once again, quantum mechanics was vindicated, and local realism took another hit.

Signal versus Noise

Worse than violating relativity, nature seems to have just destroyed the very notion of causality.

To preserve it, consider just for a moment, just for argument's sake, that all observers can actually agree on which event is the cause. This is not unreasonable, for the left/right ordering of spacelike events is a relativistic invariant. If there was some way to identify either the right or the left event as the cause, all observers could agree on this, and the temporal ordering would be ignored; it would be supplanted in such cases by left/right ordering (or right/left ordering, depending on the experimental details). Basically, this is just a variation on the theme of situation dependent absolute frames of reference pioneered by Surez & Scarini.

Now some observers will see this cause as occurring at many times the speed of light (finite speed); these are the observers for whom the cause event was earlier in time than the effect event. One special observer will see the cause as instantaneous (infinite speed); this is the observer for whom the two events happened simultaneously. But one group of observers, they will see the cause as propagating backwards in time, literally a negative speed (faster than infinity); these are the observers for whom the cause event happened later than the effect event.

Now the specter of time travel has shown up.

Yet this is not the end of the perplexing elements. Consider this, is a signal really being sent? There are only two choices for each measurement – which basis. If both measurements are in the same basis, then the results are correlated, but this can only be confirmed by comparing the two lists; and getting them together is an act that will happen only at sub light speeds. Each list, spacelike isolated from each other, is completely random. If the measurement bases are different, there is no correlation, and each list is individually still completely random. In both cases, each observer's list is absolutely random. So, no superluminal *signal* is possible; only the *noise* is correlated.

This is scant comfort. Relativity and quantum mechanics are conceptually, philosophically, at odds with one another. Somehow, nature has managed to invoke randomness to hide their incompatibility.

What is going on?

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[i] H. Zbinden, J. Brendel, W. Tittel & N. Gisin, "Experimental Test of Relativistic Quantum State Collapse with Moving Reference Frames."