Chapter 13

“12 Light-Seconds”

Credit – Fox Trot: “Football Pattern” (need permission)

Invariants

Nature seems determined to teach us that we don’t fully understand something until we look at it from multiple perspectives. Given two correlated events, X and Y, with X before Y, one might be tempted to conclude that X caused Y, but not if they are spacelike separated. Different observers will not agree on their temporal ordering. To really understand relativistic situations, they must be considered from multiple frames of reference.

The situation in quantum mechanics is much the same, where a system can be described in an infinity of possible bases. If a quantum particle is in a maximal superposition in one basis, then it will be in a classical state in an orthogonal basis. To really understand quantum situations, they must be considered from multiple bases. This is particularly true for entangled particles.

Each domain, however, contains a clue about how to understand what is going on underneath the hood; there will be an invariant. In relativity, it is the interval, the Minkowski metric. Quantum mechanics has invariants as well, though they tend to be little less easy to summarize with a single taxonomical term.

Does relativity contain other invariants?

The Thought Experiment

Our vehicle will be a thought experiment, the 12 light-second thought experiment, which found its analog in the previous discourse.

Consider a central source of light pulses that are emitted in opposite directions on one second intervals. In each direction there is a detector, 6 light-seconds away (5 times the distance to the moon). The separation between the two detectors is therefore 12 ls ('ls' as an abbreviation for light-seconds). Figure 1 shows the experimental setup as observed in the lab frame (also called the stationary frame, or the rest frame).

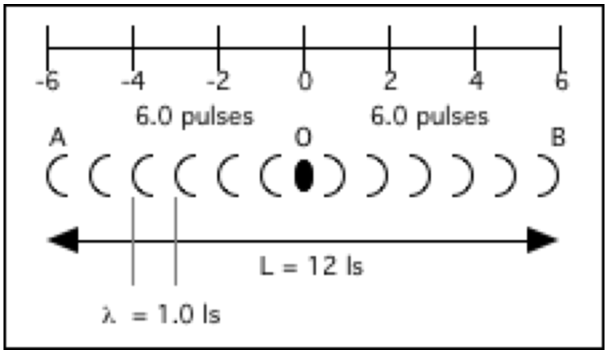

Figure 1 – **12 Light-Second Thought Experiment; Lab Frame O (0 ls/s)**: A central source of light (O) emits pulses once/second midway between two detectors (A & B) separated by 12 ls. Note that there are 12 pulses between the detectors.

Let the spatial origin of the frame of reference of the lab be the source of the light pulses and let zero time be the emission of the first pulse. Label the origin O. Label the two detectors as A (left) and B (right).

The separation between pulses is the velocity divided by the period

$$\lambda = \frac{c}{v_p} = \frac{1\ ls/s}{1/s} = 1.0\ ls \tag{1}$$

Each pulse of light is separated from its predecessor and its successor by 1 second in time and one light-second in space. There are therefore 6 pulses of light between the origin and each detector

$$\begin{gathered} n_{AO} = \frac{L}{\lambda_0} = \frac{6}{1.0} = 6 \\ n_{OB} = \frac{L}{\lambda_0} = \frac{6}{1.0} = 6 \end{gathered} \tag{2}$$

for a total of 12 pulses in flight at any time. So obvious that it hardly bears mentioning.

Frames of Reference

Now consider what a relativistic observer would see if they looked at the 12 light-second thought experiment. For maximal clarity, we'll select a relativistic observer who is moving right to left at 0.6 c, six-tenths of the speed of light (formally, u = -0.6 ls/s). We'll label both him and his frame of reference as U.

That is, this observer has a velocity (u) in the lab frame O of -0.6 ls/s. This might be called the relativistic frame, or the primed frame, as it is customary to mark variables in this frame with an apostrophe (primed).

As is always the case in relativity, the spacetime coordinates of events are not invariants, they change with one's point of view, they change with the reference frame. Therefore, the spatial and temporal separation of the light pulses is not a relativistic invariant either; U sees the scene rather differently than O does. To describe what U sees, we'll need some math. We'll derive each difference one-by-one, a straightforward application of the equations of special relativity.

A common element in relativity is the Lorentz factor, usually given the symbol gamma (γ). We compute it first as it will be needed repeatedly.

$$\gamma = \frac{1}{\sqrt{1-u^2}} = \frac{1}{\sqrt{1-0.6^2}} = 1.25 \tag{3}$$

It's a dimensionless number (no units) and for U it is equal to 1.25. We'll use it to compute the next two differences, length contraction and time dilation.

U sees different distances between the source and the detectors (and from detector to detector), because of length contraction. The equation is simple, and substitution provides the answer.

$$L' = \frac{L}{\gamma} = \frac{12}{1.25} = 9.6\ ls \tag{4}$$

In the discourse the second track was shorter than the first one – length contraction.

U also sees time as running slower because of time dilation. This equation is also simple (note how gamma has moved), and substitution provides the answer. The pulse rate, the rate at which the light pulses are being emitted, has slowed.

$$t' = \gamma t = 1.25 \cdot 1.0 = 1.25\ s \tag{5}$$

The next ways U sees this situation differently, are in the color of the pulses and their separation. The Doppler shift is the familiar change in pitch when the relative motion between a source of waves and a detector changes. When the race car zooms by, it's pitch drops; that's the Doppler shift for sound waves.

Light is also a wave; its frequency is revealed to our eyes as color. Blue has a higher frequency than red. However, unlike for sound, the Doppler equation for light (for any electromagnetic wave) also requires the Lorentz factor. The photons in the light pulses from O to B are headed toward U, so they get blue shifted by a factor of 2.

$$\left(\frac{\nu\prime}{\nu}\right)_b = \gamma(1-u) = 1.25(1+0.6) = 2.0 \tag{6}$$

while the photons in the light pulses from O to A are headed away from U, so they get red shifted by a factor of one-half.

$$\left(\frac{\nu'}{\nu}\right)_r = \gamma(1+u) = 1.25(1-0.6) = 0.5 \tag{7}$$

The simple factors ‘2.0’ and ‘0.5’ are the result of the careful choice of U’s velocity as being 0.6.

U sees the separation between the pulses differently as well.

$$\begin{aligned} \lambda_b &= \lambda_0\left(\frac{\nu}{\nu'}\right)_b = \frac{1}{2.0} = 0.5\ ls \\ \lambda_r &= \lambda_0\left(\frac{\nu}{\nu'}\right)_r = \frac{1}{0.5} = 2.0\ ls \end{aligned} \tag{8}$$

Just the inverse of the Doppler shifts.

U doesn’t even see the same number of pulses between the origin and each detector.

$$\begin{aligned} n_b &= \frac{L/2}{\lambda_b} = \frac{4.8}{0.5} = 9.6 \\ n_r &= \frac{L/2}{\lambda_r} = \frac{4.8}{2.0} = 2.4 \end{aligned} \tag{9}$$

There are 9.6 blue pulses but only 2.4 red pulses.

Figure 2 shows the scene as seen by U. From U’s point of view, from his relativistic frame of reference, nearly everything is different.

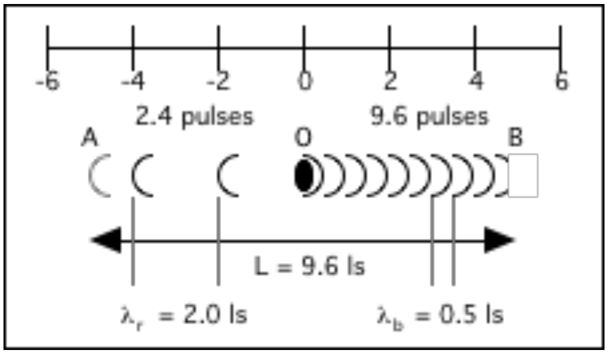

Figure 2 – **12 Light-Second Thought Experiment; Relativistic Frame U (-0.6 ls/s)**: The situation as seen by an observer moving at a relativistic velocity of -0.6

ls/s. Distance, pulse rate, color, separation, and origin to detector number of pulses; all different. But the total number of pulses between the detectors is still 12.

But remarkably, U does agree on the total number of pulses between A & B

$$n = n_{AO} + n_{OB} = n_r + n_b = 2.4 + 9.6 = 12 \tag{10}$$

which turns out to be an invariant.

As an invariant, it is independent of the reference frame; it is not a consequence of the relativistic velocity being 60% of the speed of light. That number was chosen for pedagogical reasons (makes the math more intuitive).

It is worth the investment in formality to prove this.

Consider two detectors, A and B, separated by a distance L with a dual photon source halfway between at O. O emits pairs of photons toward the detectors at regular intervals. The number of photons between the source and each detector is just the distance from source to detector divided by the separation distance between the blue and red-shifted photons.

$$n_{AB} = \frac{D_{AO}}{\lambda_r} + \frac{D_{OB}}{\lambda_b} \tag{11}$$

In the lab frame this is just

$$n_{AB} = \frac{L/2}{\lambda_0} + \frac{L/2}{\lambda_0} = \frac{L}{\lambda_0} \tag{12}$$

In the relativistic frame, accounting for the Doppler shifts, length contraction, and time dilation

$$n'_{AB} = \frac{L/2\gamma}{\lambda_0/\gamma(1+u)} + \frac{L/2\gamma}{\lambda_0/\gamma(1-u)} \tag{13}$$

Simplifying,

$$n'_{AB} = \frac{L}{2\lambda_0}[(1+u) + (1-u)] \tag{14}$$

$$n'_{AB} = \frac{L}{\lambda_0} = n_{AB}$$

which is exactly the same as for the rest frame. Therefore, the number of photons between A and B is a relativistic invariant.

The Orchard Metaphor

Imaging waking up in a forest, unsure of how you got there. At first it appears wild, unpredictable, dark, and dangerous, but a turn of the head and behold, all the trees line up. You are not in a forest, the adrenalin ebbs away, you are in an orchard, planned, planted and with a purpose. You can now make predictions about what you will see.

The above analysis reveals what U observes in his present. In a standard spacetime diagram, time is up in seconds (vertical axis) distance is sidewise in light-seconds (x-axis). Photons travel at 1 ls/s so their world lines are always at 45 degrees, in all reference frames. Now it would be natural for U to represent his observations by taking snapshots, say on the emission of every pulse; that is, every 1.25 seconds. It leads to a busy chart, shown in Figure 3. Let's tease its elements apart one at-a-time.

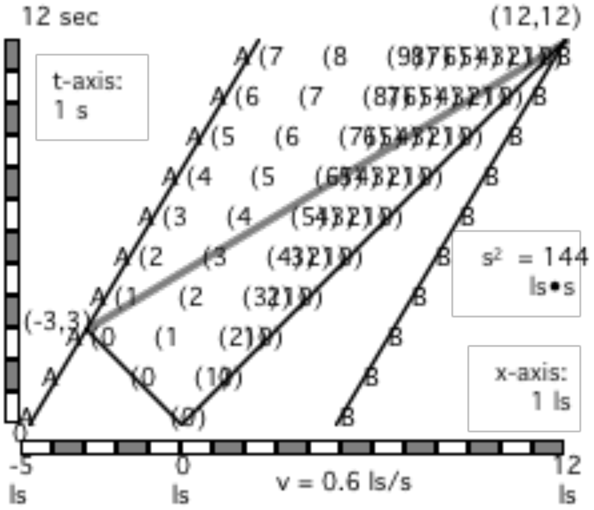

Figure 3 – **Snapshots Spacetime Diagram for U**: This spacetime diagram shows the light cone for the first pulse (0), the world lines of the detectors A & B, and the length contraction of their separation (9.6 ls) as seen in the U frame of reference. It also shows the location of every pulse on intervals of 1.25 seconds, plus the spacetime events of detection of pulse 0 (-3, 3) for A, and (12, 12) for B. The gray line connecting these events is called the symmetric spacetime interval, which is an invariant (144 in this case) between the lab and relativistic frames of reference.

Figure 3 shows the situation U sees as a spacetime diagram with snapshots taken on every pulse. Note the Poincaré rotation of the world lines of the detectors (0.6), A on the left, B on the right, the length contraction of the distance between them (9.6 ls), and the time dilation on the pulse frequency (1.25 s). Recall that from U's point of view the pulses of light are emitted every 1.25 seconds. The world lines for the photons in pulse 0 are shown, they are the diagonal lines at 45 degrees; the light cone. A snapshot of the location of every pulse (labeled from zero) as each new pulse is emitted is shown. The red and blue shifts are clearly evident as the distance between the pulses.

Zero coordinates are at the emission of the first pulse (0) at the location of the source. Successive emissions of light pulses follow the implied world line of the source (not shown), which stays halfway between A and B. When the first pulse is finally detected at B, U observes a total of 12 pulses between A and B. Note that the detection of pulse 0 at A occurs much earlier than its detection at B. From U's point of view, A has been racing towards pulse 0, closing the distance, while B has been racing away from it, opening the distance. Given that he measures the speed of light to still be *c*, it is obvious that A will receive its pulse prior to B.

To determine the spacetime coordinates of these events, all we need are the Lorentz transformations between reference frames. The spacetime coordinates of the first detection of pulses in the rest frame are (-6, 6) at A and (6, 6) at B. Therefore, U observes the first detection at A, at these spacetime coordinates.

$$\begin{aligned} x_A &= \gamma(x-ut) = 1.25(-6-(-0.6)\cdot 6) = -3.0\,ls \\ t_A &= \gamma(t-ux) = 1.25\big(6-(-0.6)\cdot(-6)\big) = 3.0\,s \end{aligned} \tag{15}$$

and the first detection at B, at spacetime coordinates

$$\begin{aligned} x_B &= \gamma(x-ut) = 1.25(6-(-0.6)\cdot 6) = -12.0\,ls \\ t_B &= \gamma(t-ux) = 1.25\big(6-(-0.6)\cdot(6)\big) = 12.0\,s \end{aligned} \tag{16}$$

which are marked on the spacetime diagram in Figure 3.

The spacetime interval connecting these two events is of course the same in both reference frames.

$$\begin{gathered} s^2 = (\Delta x)^2 - (\Delta t)^2 \\ s_O^2 = \big(6-(-6)\big)^2 - (6-6)^2 = 144 \\ s_U^2 = \big(12-(-3)\big)^2 - (12-3)^2 = 144 \end{gathered} \tag{17}$$

This spacetime interval is shown as the gray line. Since the value of *s* is a real number, it represents a spacelike connection between A and B. Since the source remains at the center of this interval, we will call it a Symmetric Spacetime Interval (SSI). Busy, busy spacetime diagram – looks like a forest, not an orchard.

While Figure 3 shows the standard use of a spacetime diagram, it gives undue emphasis to the passage of time over the extent of space. This is implicit in the sequence of snapshots of pulse locations on every time step. Unlike the previous figure however, Figure 4 does not present sequential temporal snapshots of the pulses, but rather locates each along the symmetric interval, the spacetime interval that connects the detection events. Note how the worldline of the source symmetrically bisects the symmetric interval. We choose to take it just as pulse 6 is emitted. The world line of the source has also been added; note that it bisects the symmetric interval right at the emission of pulse 6.

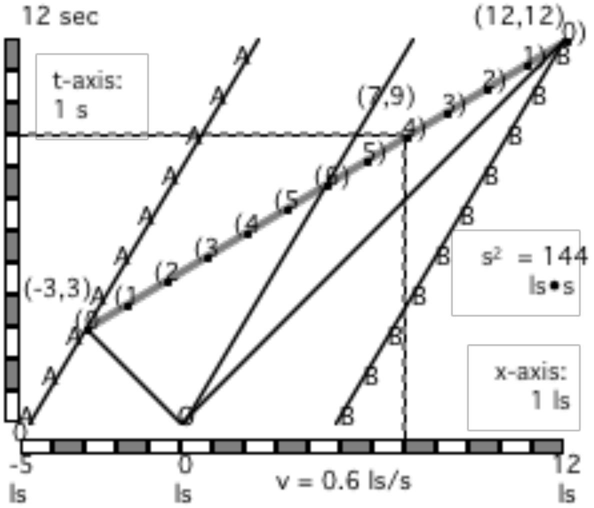

Figure 4 – **Symmetric Spacetime Interval (SSI) Spacetime Diagram for U**: This spacetime diagram shows the light cone for the first pulse (0), the world lines of the source O, and of the detectors A & B, as well as the length contraction of their separation (9.6 ls) as seen in the U frame of reference. It also shows the spacetime coordinates of the detection of pulse 0 at A (-3, 3) and at B (12, 12). It locates the pulses along the symmetric interval instead of in the plane of simultaneity. It also shows the spacetime coordinates of the 4th pulse on the way to B.

As can be readily seen, the pulses space themselves out evenly along the symmetric interval, 6 between A and O, and 6 between O and B. From this diagram, their conventional spacetime coordinates can be derived for any observer, and any observer can derive their locations along the symmetric spacetime interval from their own observations.

To demonstrate this, consider pulse 4 moving toward B just when the 6th pulse is being emitted. In the rest frame, its spacetime coordinates are (2, 6). Therefore, in the U frame

$$\begin{aligned} x_{4B} &= \gamma(x - ut) = 1.25(2 - (-0.6) \cdot 6) = 7.0\ ls \\ t_{4B} &= \gamma(t - ux) = 1.25(6 - (-0.6) \cdot 2) = 9.0\ s \end{aligned} \tag{18}$$

which puts the event right on the symmetric spacetime interval (shown with the dotted lines).

The symmetric spacetime interval shows how the pulses line up in a neat evenly spaced row. The mental image here is that the symmetric interval connects the two EPR entangled photons as they move. As they separate it lengthens. Along this interval, the pulses are evenly spaced. In the reference frame where the photons have the same color (frequency), the symmetric interval is horizontal, is defines the plane of simultaneity.

All observers can compute the symmetric spacetime interval, all of them can convert to the plane of simultaneity it represents. Any *cause* along this interval will be perceived in that reference frame as *instantaneous*. The symmetric spacetime interval defines a privileged reference frame, not universal, it is determined by the parameters of the situation, but it is *unique*, all can agree on it, all can compute it.

This opens an interesting possibility. Either the detections happen simultaneously, or one happens before the other, unambiguously. The freedom of spacelike causes to thwart temporal ordering has a loophole.

Welcome to the orchard.

<u>This overcomes the privileged observer objection.</u>